Benha University
Faculty Of Engineering at Shoubra

ECE 122
Electrical Circuits (2)(2016/2017)
Lecture (3)
Parallel Resonance

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Remember

Series Resonance

1. $\omega_o = \frac{1}{\sqrt{LC}}$, $f_o = \frac{1}{2\pi\sqrt{LC}}$

2. $Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$ → total series Res.

   $Z = R$ (at resonance).

3. $BW = \omega_2 - \omega_1 = \frac{R}{L} = \frac{\omega_o}{Q}$

4. $\omega_{1\,actual} = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)} \cong \omega_o - B/2$ → from half Power

   $\omega_{2\,actual} = \frac{+R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)} \cong \omega_o + B/2$ → from half Power

   } at $Z = \sqrt{2}R$

5. $\omega_o = \sqrt{\omega_1 \omega_2}$

6. $Q = \frac{\omega_o L}{R} = \frac{1}{\omega_o R C}$

Parallel Resonance Circuit

It is usually called tank circuit

Ideal Circuits

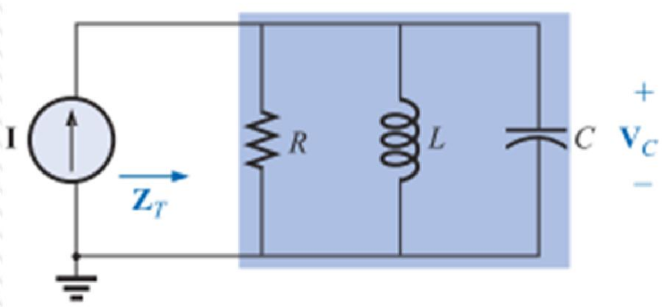


FIG. 20.21
Ideal parallel resonant network.

Practical Circuits

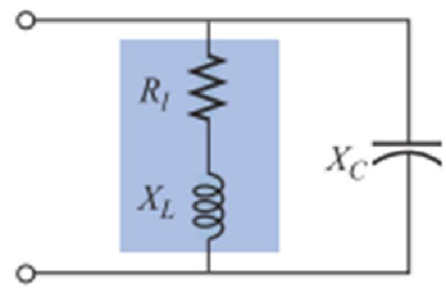
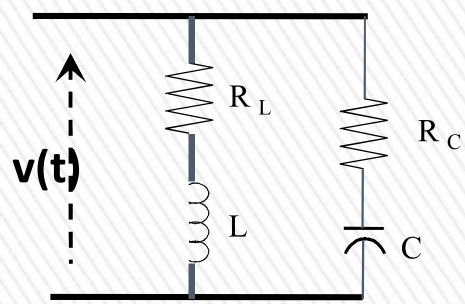


FIG. 20.22
Practical parallel L-C network.

Complex Configuration



Ideal Parallel Resonance Circuit

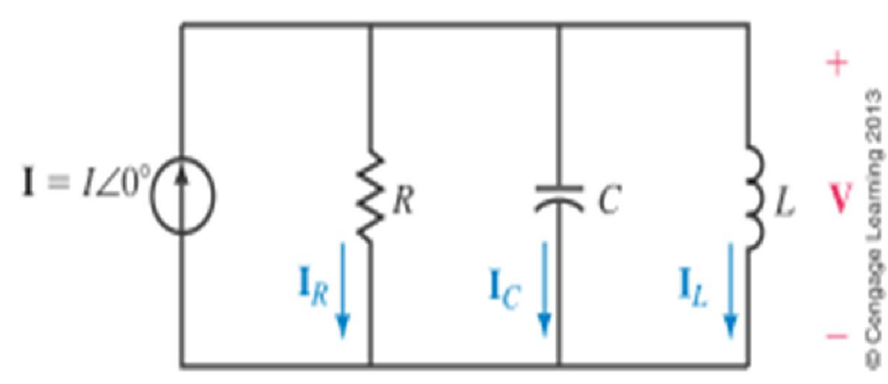
The total admittance

$$Y = Y_1 + Y_2 + Y_3$$

$$Y = \frac{1}{R} + \frac{1}{(j\omega.L)} + \frac{1}{(-j/\omega.C)}$$

$$Y = \frac{1}{R} + \frac{-j}{\omega L} + j\omega C$$

$$Y = \frac{1}{R} + j(\omega C - 1/\omega L)$$



Condition for Ideal Parallel Resonance

Resonance occurs when the imaginary part of **Y** is zero

$$\omega C - \frac{1}{\omega L} = 0$$

$$X_C = X_L.$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/s}$$

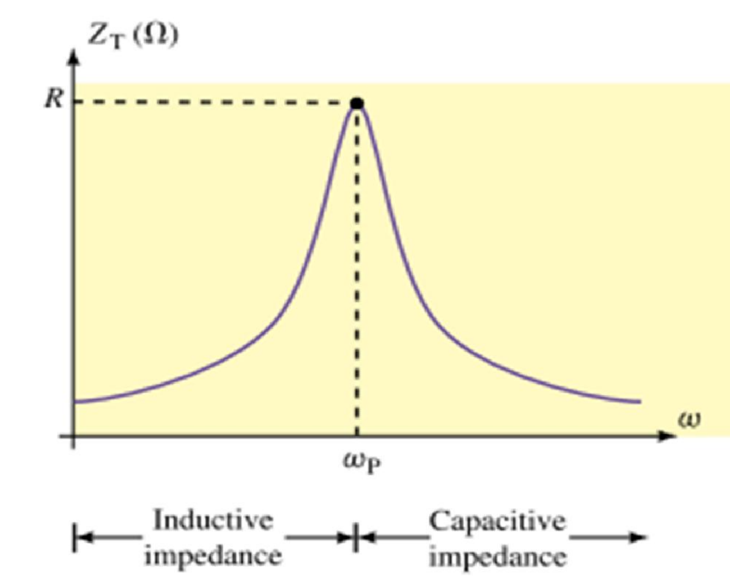
Ideal Parallel Resonance Circuit

At parallel resonance:

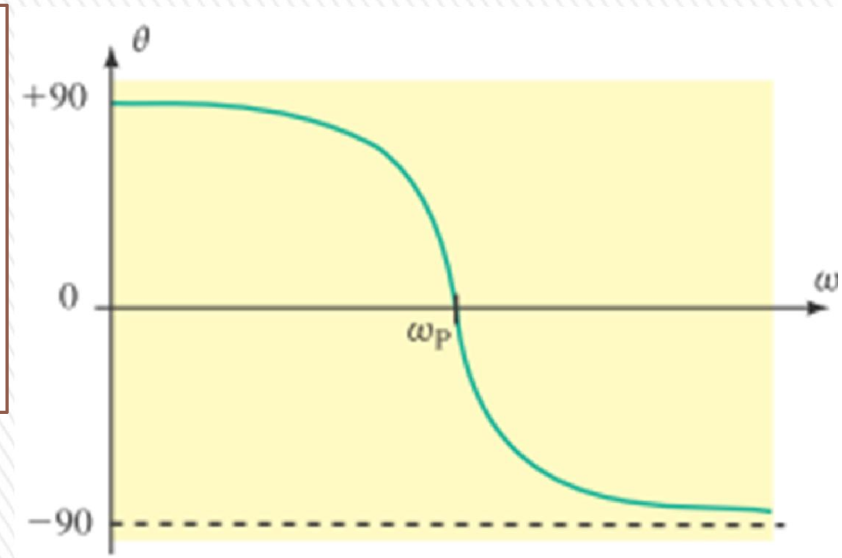
- ✓ At resonance, the admittance consists only conductance G = 1/R.
- ✓ The value of current will be minimum since the total admittance is minimum.
- ✓ The voltage and current are in phase (Power factor is unity).
- ✓ The inductor reactance and capacitor reactance canceled, resulting in a circuit voltage simply determined by Ohm's law as:

$$\mathbf{V} = \mathbf{I}R = IR\angle 0^\circ$$

- ✓ The frequency response of the impedance of the parallel circuit is shown



exactly opposite to that in series resonant circuits,



Ideal Parallel Resonance Circuit

The Q of the parallel circuit is determined from the definition as

$$Q_P = \frac{\text{reactive power}}{\text{average power}} = \frac{V^2/X_L}{V^2/R}$$

$$Q_P = \frac{R}{X_{LP}} = \frac{R}{X_C}$$

Reciprocal of series case

The current

$$\mathbf{I}_R = \frac{\mathbf{V}}{\mathbf{R}} = \mathbf{I}$$

$$\mathbf{I}_L = \frac{\mathbf{V}}{X_L\angle 90^\circ} = \frac{V}{R/Q_P}\angle -90^\circ = Q_P I\angle -90^\circ$$

$$\mathbf{I}_C = \frac{\mathbf{V}}{X_C\angle -90^\circ} = \frac{V}{R/Q_P}\angle 90^\circ = Q_P I\angle 90^\circ$$

- ✓ The currents through the inductor and the capacitor have the same magnitudes but are 180 out of phase.
- ✓ Notice that the magnitude of current in the reactive elements at resonance is Q times greater than the applied source current.

Ideal Parallel Resonance Circuit

➢ **Parallel resonant circuit has same parameters as the series resonant circuit.**

Resonance frequency:

$$\omega_p = \frac{1}{\sqrt{LC}} \text{ rad/s}$$

Half-power frequencies:

$$\omega_1 = -\frac{1}{2RC} + \sqrt{\frac{1}{4R^2C^2} + \frac{1}{LC}} \quad \text{(rad/s)}$$

$$\omega_2 = \frac{1}{2RC} + \sqrt{\frac{1}{4R^2C^2} + \frac{1}{LC}} \quad \text{(rad/s)}$$

Bandwidth and Q-factor:

$$\text{BW} = \omega_2 - \omega_1 = \frac{1}{RC} \quad \text{(rad/s)}$$

$$\text{BW} = \frac{\omega_P}{Q_P} \quad \text{(rad/s)}$$

$$\text{BW} = \frac{\omega_P}{R(\omega_P C)} = \frac{X_C}{R}\omega_P$$

$$Q = \frac{\omega_0}{B} = \omega_0 RC = \frac{R}{\omega_0 L}$$

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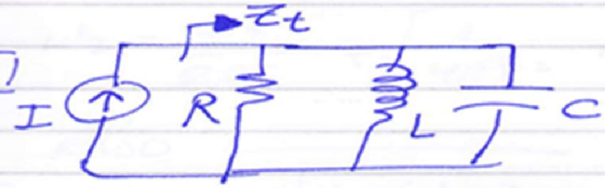
Ideal circuit

[1]



[2] $X_L = X_C$ at Resonance

$\therefore \omega_p = \frac{1}{\sqrt{LC}}$ rad/s

[3] $Y = \frac{1}{Z} = Y_1 + Y_2 + Y_3$

$= \frac{1}{R} + j(\omega C - \frac{1}{\omega L})$

[4] at Resonance $G = \frac{1}{R}$ ← admittance

& V, I in phase.

[5] $Q_p = \frac{R}{X_C} = \frac{R}{X_L} = \frac{\omega_p}{BW}$

$= R/\omega L = \omega RC$

[6] $BW = \omega_2 - \omega_1 = \frac{1}{RC}$ rad/s

$= \omega_p / Q_p$

[7] $\omega_1 = \frac{-1}{2RC} + \sqrt{\frac{1}{4R^2C^2} + \frac{1}{LC}}$

$\omega_2 = \frac{+1}{2RC} + \sqrt{\frac{1}{4R^2C^2} + \frac{1}{LC}}$ rad/s

Note for Midband

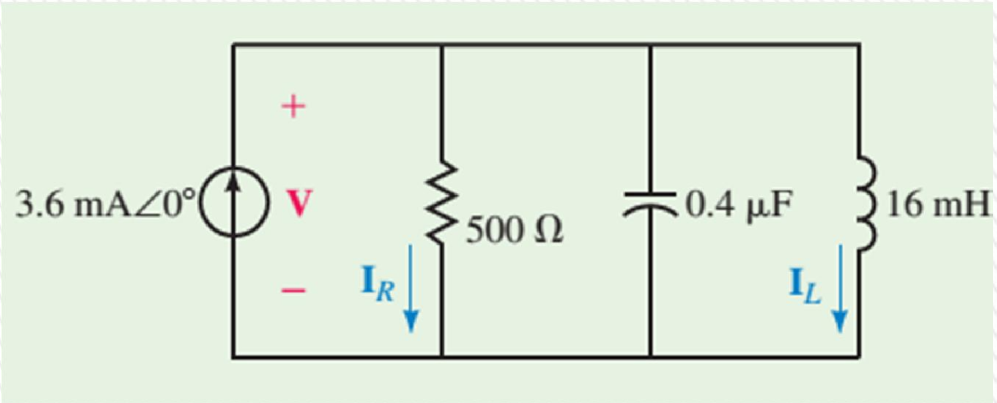
$\omega_1 = \omega_p - B/2$

$\omega_2 = \omega_p + B/2$

Example

Example

Consider the circuit shown in Figure



a. Determine the resonant frequencies, ω_P(rad/s) and f_P(Hz) of the tank circuit.

b. Find the Q of the circuit at resonance.

c. Calculate the voltage across the circuit at resonance.

d. Solve for currents through the inductor and the resistor at resonance.

e. Determine the bandwidth of the circuit in both radians per second and hertz.

f. Sketch the voltage response of the circuit, showing the voltage at the half-power frequencies.

g. Sketch the selectivity curve of the circuit showing P(watts) versus ω(rad/s).

Solution

a. $$\omega_P = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(16\text{ mH})(0.4\ \mu\text{F})}} = 12.5\text{ krad/s}$$

$$f_P = \frac{\omega}{2\pi} = \frac{12.5\text{ krad/s}}{2\pi} = 1989\text{ Hz}$$

b. $$Q_P = \frac{R_P}{\omega L} = \frac{500\ \Omega}{(12.5\text{ krad/s})(16\text{ mH})} = \frac{500\ \Omega}{200\ \Omega} = 2.5$$

c. At resonance, $\mathbf{V}_C = \mathbf{V}_L = \mathbf{V}_R$, and so

$$\mathbf{V} = \mathbf{IR} = (3.6\text{ mA}\angle 0^\circ)(500\ \Omega\angle 0^\circ) = 1.8\text{ V}\angle 0^\circ$$

d. $$\mathbf{I}_L = \frac{\mathbf{V}_L}{\mathbf{Z}_L} = \frac{1.8\text{ V}\angle 0^\circ}{200\ \Omega\angle 90^\circ} = 9.0\text{ mA}\angle -90^\circ$$

$$\mathbf{I}_R = \mathbf{I} = 3.6\text{ mA}\angle 0^\circ$$

e. $$\text{BW(rad/s)} = \frac{\omega_P}{Q_P} = \frac{12.5\text{ krad/s}}{2.5} = 5\text{ krad/s}$$

$$\text{BW(Hz)} = \frac{\text{BW(rad/s)}}{2\pi} = \frac{5\text{ krad/s}}{2\pi} = 795.8\text{ Hz}$$

f. The half-power frequencies are calculated from Equations 21–48 and 21–49 since the Q of the circuit is less than 10.

$$\begin{aligned}\omega_1 &= -\frac{1}{2RC} + \sqrt{\frac{1}{4R^2C^2} + \frac{1}{LC}} \\ &= -\frac{1}{0.0004} + \sqrt{\frac{1}{1.6\times 10^{-7}} + \frac{1}{6.4\times 10^{-9}}} \\ &= -2500 + 12\,748 \\ &= 10\,248\text{ rad/s}\end{aligned}$$

Solution

$$\omega_2 = \frac{1}{2RC} + \sqrt{\frac{1}{4R^2C^2} + \frac{1}{LC}}$$

$$= \frac{1}{0.0004} + \sqrt{\frac{1}{1.6 \times 10^{-7}} + \frac{1}{6.4 \times 10^{-9}}}$$

$$= 2500 + 12\,748$$

$$= 15\,248 \text{ rad/s}$$

The resulting voltage response curve is illustrated in Figure 21–32.

g. The power dissipated by the circuit at resonance is

$$P = \frac{V^2}{R} = \frac{(1.8 \text{ V})^2}{500\ \Omega} = 6.48 \text{ mW}$$

The selectivity curve is now easily sketched as shown in Figure 21–33.

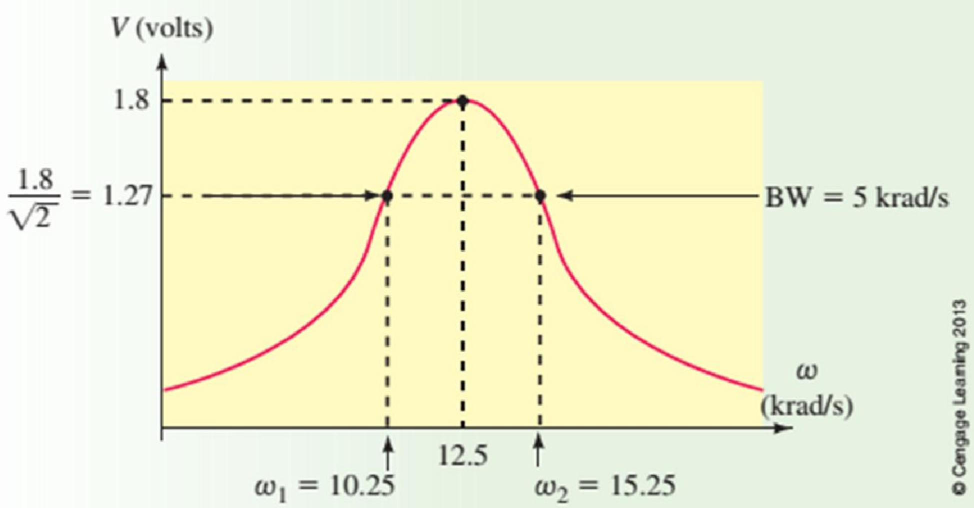


FIGURE 21–32

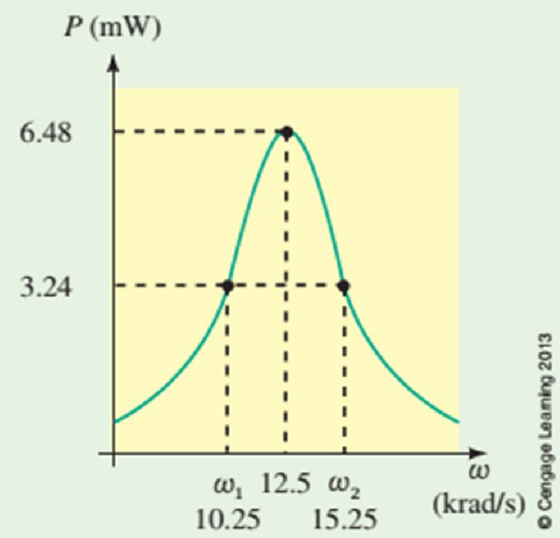


FIGURE 21–33

Thank You